An approach to 3D NURBS modeling of complex fault network considering its historic tectonics*

ZHONG Denghua**, LIU Jie and LI Mingchao

(School of Civil Engineering, Tianjin University, Tianjin 300072, China)

Received August 17, 2005; revised December 9, 2005

Abstract Fault disposal is a research area that presents difficulties in 3D geological modeling and visualization. In this paper, we propose an integrated approach to reconstructing a complex fault network (CFN). Based on the non-uniform rational B-spline (NURBS) techniques, fault surface was constructed, reflecting the regulation of its spatial tendency, and correlative surfaces were enclosed to form a fault body model. Based on these models and considering their historic tectonics, a method was put forward to settle the 3D modeling problem when the intersection of two faults in CFN induced the change of their relative positions. First, according to the relationships of intersection obtained from geological interpretation, we introduced the topological sort to determine the order of fault body construction and rebuilt fault bodies in terms of the order; then, with the disposal method of two intersectant faults in 3D modeling and applying the Boolean operation, we investigated the characteristic of faults at the intersectant part. An example of its application in hydropower engineering project was proposed. Its results show that this modeling approach can increase the computing efficiency while less computer memory is required, and it can also factually and objectively reproduce the CFN in the engineering region, which establishes a theoretical basis for 3D modeling and analysis of complex engineering geology.

Keywords: fault network, NURBS technique, historic tectonics, topological sort, 3D modeling.

Fault is the part in rock mass where the mechanical strength is comparatively weak. Fault can affirmatively cause discontinuity and inhomogeneity of rock mechanics, which often induces unstability and leakage in the engineering rock mass. Therefore, fault structure is one key geofactor that should be of great importance in 3D geological model. Methodologies for computer simulation of faults by 3D model have been studied for many years, and some methods of modeling faults by 3D simulation have been proposed. However, due to the complexity and uncertainty of fault in geological surfaces and volumes, up to now, there have been no perfect software tools or easy-to-handle solutions.

In reviewing the current techniques for 3D modeling of faults, some most popular techniques applied are those described by Breuning[1], Egan et al.[2], Kreuseler[3], Zehnder and Allmendinger[4], in which, based on geophysical data and observations from study areas, fault surfaces can be reasonably formed. However, these methods require abundant information to be available to build models, and they are only effective for dealing with single valued surfaces. With mathematical description, mathematical model of fault[5] can be established to effectively describe multivalued surfaces, such as reverse faults. However, because this model employs a combination of multiple planes to approximate the complicated fault surface, they cannot validly extrapolate the continuation of fault beyond the sampled data. To model complicated geologic bodies such as continuous object termination and continuous object breaking, a modeling algorithm, based on the improved triangular prism model, has been put forward[6]. Zhu et al.[7] and Meng[8] adopted the triangulated irregular network (TIN) to describe faulted structures. Although both the above approaches might well grasp the regulation of fault spatial distribution, there exist some limitations, for instance, large computer memory is required, operations are inflexible and the computing speed is lower.

Recent research on fault disposal mostly focuses on construction of a single fault or only two faults intersecting with each other, but little attention has been paid to modeling complex fault network (CFN). Wu[9] presented a mix-mesh model to build the complicated structure of fault network. Without considering the historic development of a fault and the rela-

---

* Supported by National Natural Science Foundation of China (Grant Nos. 50479048, 50525927, 50539120 and 50479048)

** To whom correspondence should be addressed. E-mail: dzhong@tju.edu.cn

tionship of faults intersecting, the fault network modeled only treats its geometrical configuration, but cannot reflect the tectonic mechanism of faults. That is to say, it cannot truly express the characteristic of location where faults intersect.

In a word, to rebuild CFN, not only should the fault body be constructed effectively and accurately, but also the critical issue of two intersectant faults must be objectively disposed of. To solve the above problems, an integrated approach to modeling CFN in 3D is proposed, which combines the non-uniform rational B-spline (NURBS) techniques and topological sort for directed acyclic graph (DAG). Finally, this approach is successfully applied to Huizhou pump storage hydroplant engineering.

1 NURBS modeling of fault body

NURBS technique is the expression standard in STEP (ISO, 1991) for free-form curves and surfaces. With the development of computer aided geometry design (CAGD), NURBS technique has rapidly developed and been widely applied.

Due to the complexity and irregularity of fault configuration, NURBS technique was utilized to model complicated fault bodies in this study. This approach is computationally efficient, requires lower computational memory and data processing is more valid[10]. In addition, NURBS technique is very effective in describing the multivalued surface, and modeling space uniqueness and geometry inalterability of geological structures[11].

1.1 Representation of NURBS surface

NURBS technique mathematically constructs complex geological structures with smooth continuous curves and surfaces using only sparse control points. Problems related to polynomial interpolation, such as large amplitude folds resulting from interpolating adjacent subparallel control points with high-order polynomials, are avoided in this technique[12—14].

A NURBS surface with control points P_{ij} $(0 \leqslant i \leqslant m, 0 \leqslant j \leqslant n)$ can be defined as

$$S(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}P_{ij}N_i^k(u)N_j^l(v) \Big/ \sum_{i=0}^{m}\sum_{j=0}^{n} w_{ij}N_i^k(u)N_j^l(v), \tag{1}$$

where w_{ij} is the corresponding weight of P_{ij}; $N_i^k(u)$ and $N_j^l(v)$ are the normalized B-spline base functions of orders k and l, defined over knot vector $\boldsymbol{U} = \{u_0, \cdots, u_{m+k} \mid u_i \leqslant u_{i+1}, i = 0, \cdots, (m+k-1)\}$ and $\boldsymbol{V} = \{v_0, \cdots, v_{n+l} \mid v_j \leqslant v_{j+1}, j = 0, \cdots, (n+l-1)\}$, respectively; $S_{ij}(u,v)$ represent surface segments (Fig. 1), $u \in [u_{k-1}, u_{m+1}]$, $v \in [v_{l-1}, v_{n+1}]$. Generally, k and l are set to be 3.

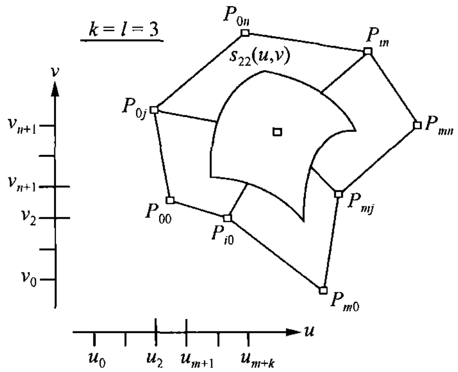


Fig. 1. Control points of NURBS surface.

1.2 Fault surface modeling using NURBS

Taking a fault as a simple example, its tectonic surface can be mathematically described according to Eq. (1). The surface modeling procedure is stated as follows:

(i) Based on the data collected from drill cores and interpretation maps, curvilinear vectors of this surface in $\boldsymbol{u}$, $\boldsymbol{v}$ directions can be derived.

(ii) The control points of each vector are determined by the back-calculation method[15]. A NURBS surface can be fitted by a few given control points. However, the given data are actual points of geological surface and they cannot be used to construct the NURBS surface directly. To get control points, the actual data must be back-calculated twice in $\boldsymbol{u}$, v directions in turn[16].

(iii) All initial weights, w_{ij}, are set to 1 initially, the corresponding NURBS surface will be fitted and constructed by interpolating control points, in which the mesh matrixes of the u, v directions can be given interactively, such as 160 by 80. The surface is fitted by repeatedly adjusting these weights from 0.1 to 10, according to the importance of the corresponding drill cores.

(iv) Finally, the surface boundary is defined and clipped. The NURBS mesh surface without rendering for this example is shown in Fig. 2.

1.3 Fault body modeling

Fault developing in rock mass is always not a

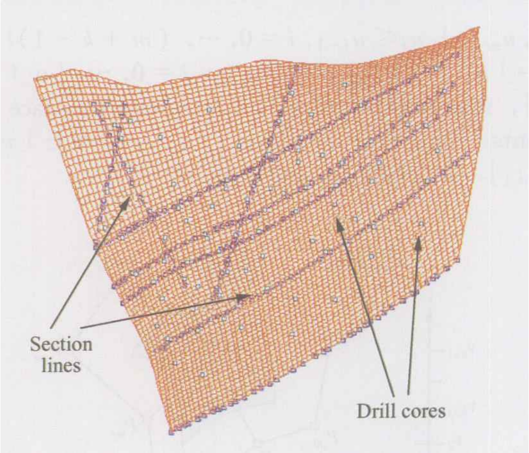


Fig. 2. A fault surface with control points.

single surface, but often appears as a zone of fracture with some width. It may be made up by subparallel fault surfaces[17]. Therefore, Fault in geometric space can be described as a body with outline, and the fault surface is its key component. The model of fault body can be described by the following definition:

$$f = S_u \cup S_d \cup S_c. \tag{2}$$

In Eq. (2), $S_i = s(P_i)(i = u, d, c)$, where s is the constructed function in NURBS technique, and S_i is the aggregate of NURBS surface; $P_i = \{p \mid p \in R^3\}$ $(i = u, d)$, and P_i is the 3D point aggregate which is gained through making use of discrete geophysical data by the back-calculation method; $P_c = (P_u^* \cup P_d^*)$, $P_i^*(i = u, d)$ is the 3D point aggregate in the boundary of NURBS surface S_i.

Therefore, every fault body f can be constituted by three surface aggregates, S_u, S_d and S_c, which respectively represent the upper tectonic surface, down tectonic surface and boundary surfaces.

2 Topological sort

2.1 Principle of topological sort

In practice, directed graph (DG) is usually used to describe a layout chart. Such DG, in which the vertices represent activities and the directed edges represent precedence relation between activities, is called activity on vertex network (AOV network)[18].

$G = (V, E)$ is an arbitrary AOV network, and V is the finite aggregate of vertex, E is the aggregate of edge. For arbitrary $v \in V$, $\{u \mid u \in V \cap \langle u, v\rangle \in E \cap u \neq v\}$ is called the predecessor aggregate of v, namely any vertex which precedes v and has a directed path accessing to v is a predecessor of v. Apparently, there exists no cycle in the AOV network, otherwise the corresponding activities could not work.

Given that vertex v_i is a predecessor of vertex v_j, then activity v_i precedes activity v_j, the method for gaining the linear order linking these two vertexes is called topological sort. In this paper, the topological sort is based on DAG.

Topological sort is usually applied to such fields as construction planning for project, production process design, and so on. Due to the precedence order in the fault developing process, this sort of method can be introduced to gain the linear order for fault modeling.

2.2 Algorithm of topological sort

Topological sort can be implemented in different physical storages for AOV network. Based on the adjacent matrix memory, although this algorithm is simple, the topological sort for AOV network has the disadvantage of lower efficiency of computing. Storage organization of adjacency list is better for the AOV network, because the corresponding algorithm might evidently improve the sorting efficiency. The correlative variables are defined in C++ as follows:

```
const int n;                    // n, the number of node
Struct Node                     // the structure of node in AOV network
{
    int ID;                     // sequence number of node
    int in;                     // in-degree of node
    string name;                // name of node
    Edge* out;                  // the associated out-edge
};
Struct Edge                     // edge in the AOV network
{
    int ID;                     // sequence number of node
```

```
        Edge* link;                       // the associated out-edge
    };
    Struct Nodelist                       // nodes table
    {
        NodeArray nodelist[n];            // array list of nodes
    };
```

The field *in*, representing in-degree of node, is attached to every node in the node table. Adopting adjacency list for AOV network storing, the algorithm of topological sort is generalized as follows:

Step 1. Use all nodes whose in-degree is zero to construct a chain-stored stack.

Step 2. Pop a node from the stack head, then access its list of out-edge. Traverse the nodes that these out-edges are pointed to and tally down their field in. Then, if there exist nodes which are of in-degree zero and not in the stack, singly push them into the stack.

Step 3. Repeat step 2 until the stack is null; the sort is completed.

3 3D modeling of two intersectant faults

3.1 Implementation of a fault body modeling

According to the fault body model in Eq. (2), a single fault can be rebuilt by suturing the upper tectonic surface, down tectonic surface and boundary surfaces. Hence, construction of a single fault with 3D model requires two steps: 3D visualization of fault surfaces and suturing the correlative surfaces. The former is nothing more than the construction of fault surface with NURBS technique, which has been stated in Section 1.2. And fundamentally the latter is decomposed into three steps:

Step 1. With restriction of boundary and termination disposal, two simulative fault surfaces (S_u and S_d) which follow the rule of a real fault distributing in rock mass, are gained;

Step 2. Take advantage of the edge of the fault surfaces after disposal as mentioned above to construct the boundary surfaces (S_c);

Step 3. With the premise of satisfying a given precision, apply the Boolean technique and geometrical operations to close the correlative surfaces to form a fault body.

An example of a fault body model is shown in Fig.3. However, the fault body still needs check-up and verification, such as geometry and rationality check-up, precision verification, etc. If there exists incorrectness or dissatisfaction, the body should be amended or even rebuilt.

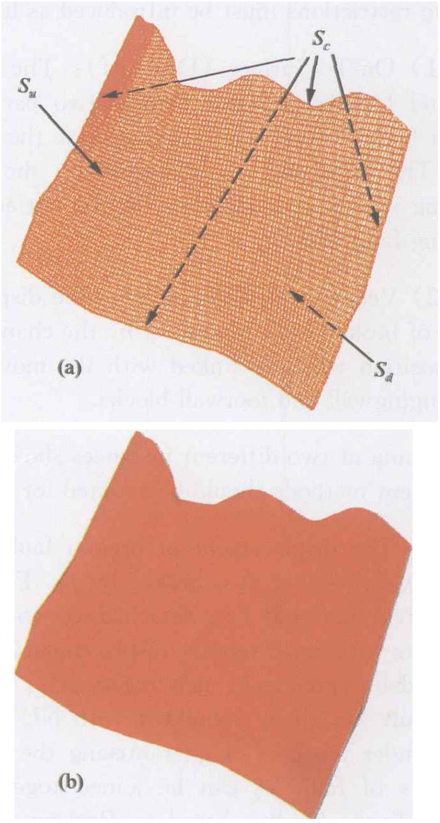


Fig. 3. A fault body model. (a) Meshing; (b) rendering scene.

3.2 Disposal of two intersectant faults in 3D modeling

In the engineering region, where the geological structure is very complicated, many faults develop in a concentrated fashion, and the relative position of fault broken by other faults often changes, as illustrated in Fig.4. From the interpreted faults attitude and tectonic characteristic, we can find that faults f_2 and f_4 developing previously are broken by faults f_1 and f_3 developing later, respectively.

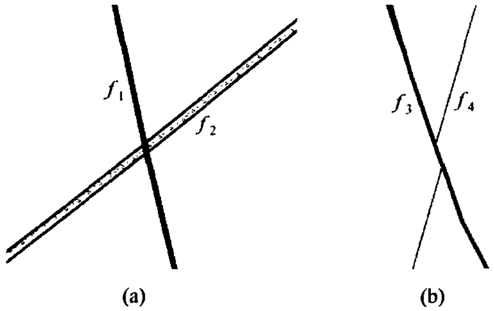


Fig. 4. Two intersectant faults in different changes of the relative positions. (a) Tiny displacement; (b) large displacement.

To model two faults that intersect with each other, two restrictions must be introduced as follows:

(1) On-To-Surface (OnTsurf): The hangingwall and footwall blocks, namely two parts of the broken fault, should be terminated at the breaking fault. That is to say, their borderline, the result of breading movement, must be on the surface of the breaking fault body.

(2) Vector-Link (VecLink): The displacement vector of broken fault can be set by the change of relative position which is linked with the movement of the hangingwall and footwall blocks.

Aiming at two different instances shown in Fig. 4, different methods should be adopted for modeling:

(1) The displacement of broken fault is tiny. Fig.4(a) shows that f_2 is broken by f_1. Fault f_1 remains integrated, but f_2 is detached to two discontinuous parts with small relative displacement. The concrete modeling process in such instances is: first, construct fault f_1; then, rebuild it with NURBS technique, under the case of guaranteeing the accuracy, two parts of fault f_2 can be joined together with NURBS lines; finally, based on Boolean operation, use fault f_1 to cut the part where faults f_1 and f_2 intersect. Thus, the ultimate model not only satisfies the precision required, but also meets the two restrictions.

(2) The displacement of broken fault is large. Fig.4(b) shows that f_3 breaks f_4. Fault f_3 remains a whole one, but f_4 is cut by two discontinuous parts with large relative displacement. In this case, if the disposal of f_4 is alike f_2, which may induce abrupt inflexion at the surface of f_4, the result does not satisfy what is looked forward to. Therefore, additional practice is adopted as follows: the two parts of f_4, corresponding to the hangingwall and footwall blocks of f_3, are separately constructed, which satisfies the restriction of OnTsurf; during the construction process, adjustment should be done to make the borderline of f_4 to be located on f_3. Thus, the restriction of VecLink is also satisfied.

The final model of two intersectant faults agrees with the tectonic movement, and also establishes a basis for further work on modeling of CFN.

4 3D modeling of CFN

In the engineering region, where the geological structure is very complicated, lots of concentrated faults develop, many intersecting with others. Thus, it is popular to develop the CFN in the geological rock mass, which induces frequent collapse and leakage in the location where two faults intersect. Therefore it is significant to truly express the characteristic of location where faults intersect.

4.1 Establishment of AOV network for fault network

Faults in CFN have their own precedence order during the developing process. That is to say, there absolutely exists no closed loop in their sequences. Hence, according to the principle of topological sort mentioned in Section 2.1, a CFN can be defined as AOV network:

$$G = (F, E), \quad F = \{f_i \mid i = 1, 2, \cdots, n\},$$
$$E = \{\langle f_i, f_j \rangle \mid f_i, f_j \in F, i, j = 1, 2, \cdots, n\},$$

where F represents the aggregate of faults in CFN; E represents the aggregate of the ordered pair among elements in F, namely the edges in the AOV network; $\langle f_i, f_j \rangle$ indicates that fault f_i appears before fault f_j, and f_j breaks f_i, or f_j terminates at f_i.

If the in-degree of a vertex is zero, then the fault represented by this vertex forms relatively early, but not absolutely ahead of the others. To determine its order of precedence, comparison with the other node of in-degree zero should be done. The final directed sequence $f_1 \rightarrow f_2 \rightarrow \cdots \rightarrow f_n$ represents the order of modeling for faults in CFN, but does not demonstrate the sequence of developing. Therefore, although there exist multi-sequence phenomena after topological sort, the space-time relationship among the faults of CFN in 3D cannot be influenced.

Fig.5 illustrates the condition of faults developing in local engineering region, and seven faults making up of a fault network. The tectonic process is interpreted from the occurrence and structural feature offaults. Therefore, it is easy to determine the precedence of any two faults. The ordered pairs gained are given below:

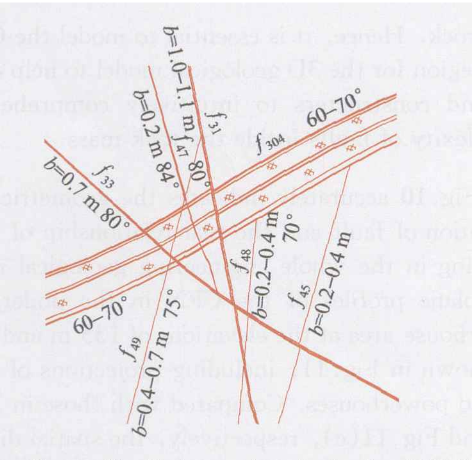


Fig. 5. Plane profile of fault network at the elevation of 135.0 m.

(f_{304}, f_{49}), (f_{304}, f_{31}), (f_{304}, f_{33}), (f_{304}, f_{47}), (f_{304}, f_{48}), (f_{304}, f_{45}), (f_{49}, f_{33}), (f_{49}, f_{47}), (f_{31}, f_{33}), (f_{33}, f_{47}), (f_{48}, f_{31}), (f_{48}, f_{33}), (f_{45}, f_{33}).

The built AOV network is shown in Fig.6.

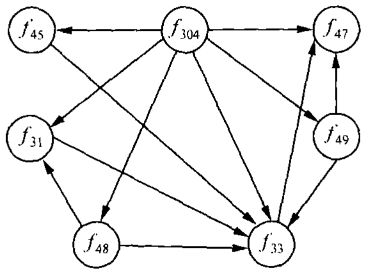


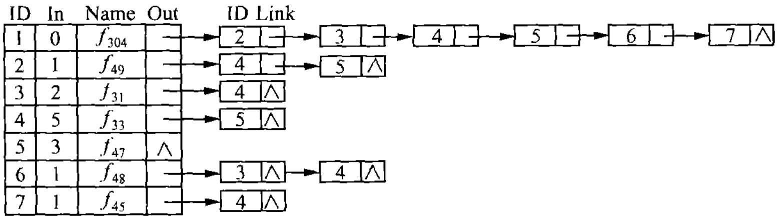
Fig. 6. The AOV network for fault network.

4.2 Topological sort of the AOV model of fault network and implementation of the final 3D model

Based on the relationship of ordered pairs, the AOV network in Fig.6 can be stored in adjacent list, and the result is shown in Fig.7.

After the AOV network is established and stored in adjacent list, it is easy to obtain the topological sequence using the topological sort algorithm. The sorting result of Fig.7 is

$$f_{304} \to f_{45} \to f_{48} \to f_{31} \to f_{49} \to f_{33} \to f_{47}.$$



ID	In	Name	Out	ID Link
1	0	f_{304}	→	2 → 3 → 4 → 5 → 6 → 7 ∧
2	1	f_{49}	→	4 → 5 ∧
3	2	f_{31}	→	4 ∧
4	5	f_{33}	→	5 ∧
5	3	f_{47}	∧	
6	1	f_{48}	→	3 → 4 ∧
7	1	f_{45}	→	4 ∧

Fig. 7. Storing the AOV network in adjacent list.

In the same 3D Euclidean space, fault body is constructed in light of the above sequence, and with the disposal of two intersectant faults, a whole fault network model is accomplished (Fig.8). Compared with Fig.5, because the historic tectonic process is considered, the 3D model built not only displays the spatial distribution of faults, but also objectively and actually reveals the topological relationship between faults in the fault network.

5 Engineering example

Huizhou Pump Storage Hydroplant, located in Luoyang Town, Boluo County, is the second pump storage hydroplant with high hydraulic pressure and large capacity, in Guangdong Province. Its total installed capacity is 2400 MW, so it is also the largest of all such hydroplants being built presently in China. This project has upper reservoir and down reservoir, and the natural fall is up to 531.0 m. Fig.9 shows the 3D geological model of the engineering region of this project.

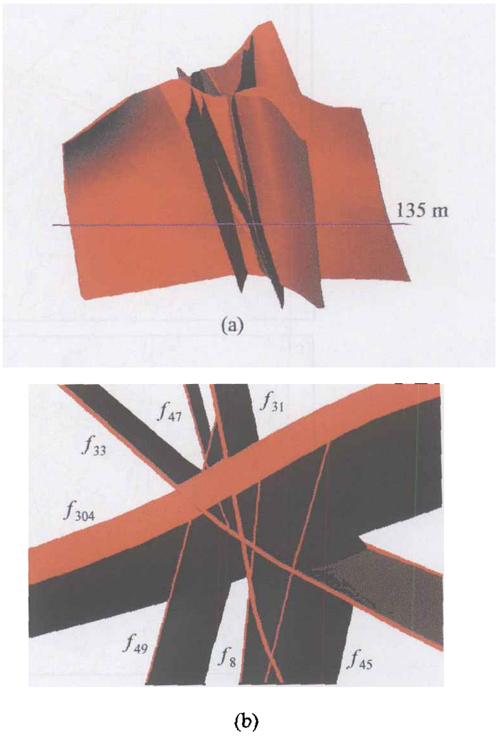


Fig. 8. 3D model of fault network. (a) A whole model; (b) plane profile at the elevation of 135.0 m.

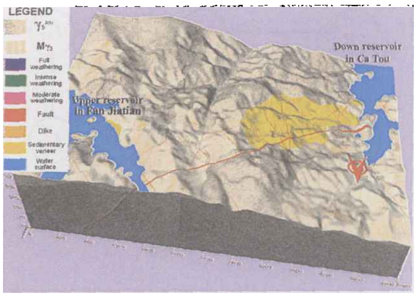


Fig. 9. 3D geological model of the engineering region of the pump-generator plant.

This hydroplant has two underground powerhouses A and B, both deep in granite mass. The size of powerhouses is 152.0 m(L)×21.5 m(W)×49.4 m(H). In so large an underground cavity, more attention should be paid to the condition of engineering geology. The trial heading exposes 67 faults in the powerhouse area. In terms of their strike, all faults can be classified into five groups, north-west(NW), north-north-west (NNW), north-east (NE), north-north-east(NNE) and nearly east-west(EW). Of the five groups, faults with the strike of NW, NNW or NE are in majority. The breaking movement between two faults occurs frequently, so all faults investigated form a large CFN. The CFN plays a decisive role in safety in the excavating process and in stabilization of wall rock. Hence, it is essential to model the CFN in this region for the 3D geological model to help designers and constructers to intuitively comprehend the complexity of faults inside the rock mass.

Fig.10 accurately indicates the geometrical configuration of fault and the real relationship of fault's breaking in the whole engineering geological region. The plane profiles of the CFN in the underground powerhouse area at the elevations of 135 m and 246 m are shown in Fig.11, including projections of underground powerhouses. Compared with those in Fig.11 (a) and Fig.11(c), respectively, the spatial distribution and local characteristic of the modeled faults displayed in Fig.11(b) and Fig.11(d) are nearly the same as the CAD data from geological exploration.

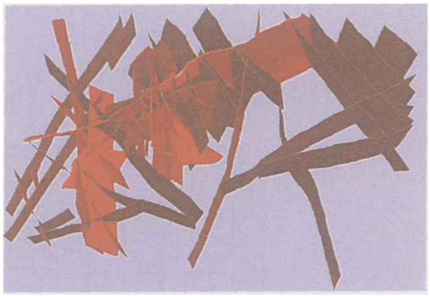


Fig. 10. 3D model of the CFN in engineering region.

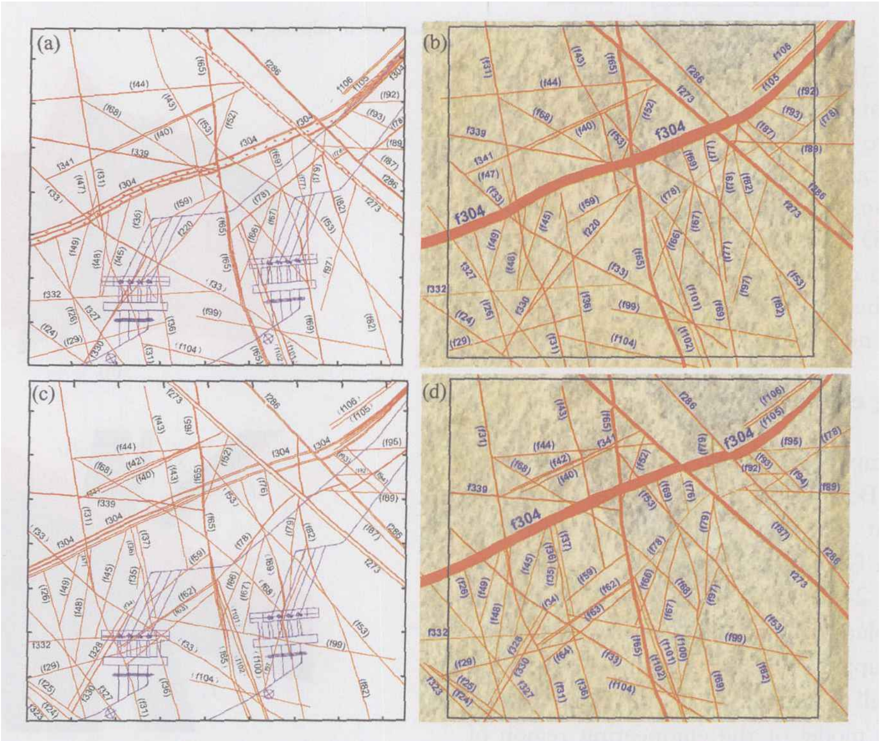


Fig. 11. The CFN model of underground powerhouse region in truncation. (a) CAD data at the elevation of 135 m; (b) rendering scene at the elevation of 135 m(with stratum); (c) CAD data at the elevation of 246 m; (d) rendering scene at the elevation of 246 m(with stratum).

6 Conclusions

With the rapid development of computer science and graphics techniques, modeling and visualization of engineering geology has been an important research area, while fault disposal is a major problem within them. This paper has presented an integrated approach to the reconstruction of CFN in 3D. First, based on the NURBS technique, the fault tectonic surface is approximated, the fault body model is rebuilt with correlative surfaces, and the topological sort is introduced to determine the order of fault construction; then, considering the disposal of two intersectant faults, the Boolean operation is used to model CFN. This approach has been successfully applied to studying the geological faults in Huizhou pump storage hydroplant engineering. Compared to the conventional 3D geological modeling technique, this approach requires less computer memory and can increase the computing efficiency. In addition, the final model not only reflects the spatial distribution of every fault body, but also reveals the characteristic of two intersectant faults in CFN, which is the result of tectonic movement. Therefore, this model is more objective and more realistic, and will be better for geological analysis in 3D.

The success of this model is mostly based on the relatively sufficient sampled data. Due to the geometric irregularity and uncertainty in geological fault surfaces and volumes, and the limited devotion of labor power and material resources, it is very difficult to gain sufficient data of geologic exploration. Hence, apart from completing the proposed theory and approach, further research will be devoted to accurately predicting the continuation of faults, which will produce the best model in conformity to the regularity of fault spatial tendency, and will bring large profit in engineering.

Acknowledgements The authors would like to thank Dr. Pete Paradiso for his help in modifying the English in this paper.

References

1 Breunig M. An approach to the integration of spatial data and systems for a 3D geo-information system. Computers & Geosciences, 1999, 25 (1): 39—48.

2 Egan S.S., Kane S., Buddin T.S. et al. Computer modeling and visualisation of the structural deformation caused by movement along geological faults. Computers & Geosciences, 1999, 25 (3): 283—297.

3 Kreuseler M. Visualization of geographically related multidimensional data in virtual 3D scenes. Computer & Geosciences, 2000, 26(1): 101—108.

4 Zehnder A.T. and Allmendinger R.W. Velocity field for the trishear model. Journal of Structural Geology, 2000, 22 (8): 1009—1014.

5 Wu Q. and Xu H. Research on the approach to modeling and visualization of geology in 3D. Science in China (Ser. D), 2004, 34 (1): 54—60.

6 Wang Z.G. and Cao D.Y. 3D modeling method for complicated geologic body based on the improved triangular prism model. Coal Geology of China, 2004, 16(1): 4—6.

7 Zhu H.H., Zheng G.P., Wu J.B. et al. Study on ground data model based on drill hole information. Journal of Tongji University (in Chinese), 2003, 31(5): 535—539.

8 Meng X.H., Wang W.M., Yao C.L. et al. The Principle and Application of Computer Aided Design to Geological Model, 1st ed. Beijing: Geological Publishing House, 2001.

9 Wu Q. and Xu H. An approach to computer modeling and visualization of geological faults in 3D. Computers & Geosciences, 2003, 29(4): 503—509.

10 Zhong D.H., Li M.C., Wang G. et al. Visualization analysis of complex geologic body with NURBS modeling. Journal of Computer-Aided Design & Computer Graphics, 2005 17(2): 285—290.

11 Li Q.Q. and Li D.R. Data structure in 3D geographic information systems. Journal of Wuhan Technical University of Surveying and Mapping, 1996, 21(2): 128—133.

12 De Paor D. G. Bézier curves and geological design. In: Structural Geology and Personal Computers. Oxford: Pergamon Press, 1996, 389—417.

13 Piegl L. and Tiller W. The NURBS Book, 2nd ed. New York: Springer, 1997.

14 Zhu X.X. Modeling Technology of Free-Form Curves and Surfaces. Beijing: Science Press, 2000.

15 Shi F.Z. Computer Aided Geometry Design and Non-Uniform Rational B-Spline (CAGD & NURBS). Beijing: High Education Press, 2001.

16 Farin G. Curves and Surfaces for Computer Aided Geometric Design, 4th ed. San Diego: Academic Press, 1997.

17 Yu H.N. and Lu H.F. The Principle of Tectonic Geology. Nanjing: Nanjing University Press, 1998.

18 Sartaj S. Data Structures, Algorithms, and Application in C++, 2nd ed. New Jersey: Silicon Press, 2004.